**Program**

**Final control for the course “Measure and Lebesgue Integrals” for the 2023/2024 academic year**

**Faculty** Mechanics and Mathematics

**Department** Mathematics

**Code and name of the educational program**: 6B05402 - Mathematics

**Name of discipline**: **Measure and Lebesgue Integrals**

**Course** \_3\_Course number\_148332\_

**Teacher:** Kaliyeva Kulyash

**Protocol and date of consideration and approval by the department** **Mathematics 09.11.2023**\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**The form of final control** in an academic discipline is oral traditional (topics include IWS , seminars)

**Platform**: no

Oral exam - the student takes an exam in the building in front of the examination committee according to the exam schedule by answering the questions on the ticket. The exam is monitored by a video camera.

**TRADITIONAL ORAL EXAMINATION – ANSWERS TO QUESTIONS.**

**Conducted offline. The exam format is synchronous.**

The process of passing an oral examination by a student involves the creation of an examination paper for the student, which must be answered before the examination committee.

**EXAMINATION REGULATIONS**

**IMPORTANT** – the exam is conducted according to a schedule that must be known to students and teachers in advance. This is the responsibility of the departments and faculty.

**STUDENT**

1. It is necessary to come to the building in advance and find an audience to take the exam.

2. Provide identification and sign the appearance form.

3. Enter according to your queue.

4. Take a ticket and prepare to answer the questions on the ticket.

5. The duration of preparation is determined by the commission (recommended 10 minutes preparation and 5 minutes response).

6. After preparation, the student defends his answers before the commission.

**List of topics for the oral final exam in the discipline**

**MODULE 1**

**Measure Theory**

 **1.**Measure of Plane Sets. Systems of Sets.

2. Measures on Semi rings. Continuation of a Measure from a Semi ring to the Minimal Ring over it.

3. Continuations of Jordan Measures

4. Measure of Plane Sets. Jordan Measures. Systems of Sets.

5 Lebesgue Continuation of Measure, Defined on a Semi ring with a Unit

**MODULE 2**

**Measurable Functions**

6. Definition and Basic Properties of Measurable Functions

7.Sequences of Measurable Functions. Different Types of Convergence

8 Direct Products of Systems of Sets and Measures

9. Expressing the Plane Measure by the Integral of a Linear Measure.

10.Integral of a Linear Measure

**MODULE 3**

**The Lebesgue Integral**

11. The Lebesgue Integral for Simple Functions

12. General Definition and Basic Properties of the Lebesgue Integral

13. The Lebesgue Integral and the Riemann Integral

**MODULE 4**

**Functions Which Are Square Integrable**

**14.** Orthogonal Systems of Functions.

15.Orthogonalisation**.** Fourier Series on Orthogonal Systems.

 16.Riesz-Fischer Theorem

17.The Isomorphism of the Spaces L2 and l2

**MODULE 5**

**The Abstract Hilbert Space.**

18.Integral Equations with a Symmetric Kernel

19.Linear and Bilinear Functionals in Hilbert Space

20.Completely Continuous Self-Adjoint Operators in H

21.Linear Equations with Completely Continuous Operators

22. Integral Equations with a Symmetric Kernel

23.Basic Properties of Hilbert Space

**Literature:**

**Main**

1. Kolmogorov, A. N., and Fomin, S. V. Measure, Lebesgue Integrals, and Hilbert Space/ Moscow State University Moscow, U.S.S.R, 1962.

2. Kolmogorov, A. N., and Fomin, S. V. Elements of the Theory of Functions and Functiorwl Arwlysis, Volume I. New York: Graylock, 1957.

3. Anh Quang Le Measure and Integration Problems with Solutions, 2013

**Additional**

1. Banach, S. Theorie des Operations Lineaires. (Monografje Matematyczne, Tom I). Warszawa, 1932.

2. Dieudonne, J. Foundations of Modern Arwlysis. New York: Academic Press, 1960. 3. Goffman, C. Real Functions. New York: Rinehart, 1953.

 4. Hall, D. W., and Spencer, G. L. Elementary Topology. New York: Wiley, 1955. 5. Hahn, H., and Rosenthal, A. Set Functions. Albuquerque: The University of New Mexico Press, 1948.

6. Halmos, P.R. Introduction to Hilbert Space. New York: Chelsea, 1951. 7. Halmos, P.R. Measure Theory. New York: Van Nostrand, 1950.

8. Hille, E. Functional Arwlysis and Semi-Groups. New York: American Mathematical Society, 1948.

 9. Kestelman, H. Modern Theorles of Integration. Oxford: Oxford University Press, 1941.

10. Munroe, M. E. Introduction to Measure and Integration. New York: AddisonWesley, 1953.

 11. Nathanson, I. P. Theory of Functions of a Real Variable. New York: Ungar, 1955. 12. Petrovskii, I. G. Lectures on the Theory of Integral Equations. New York: Graylock, 1957.

13. Riesz, F., and Sz-Nagy, B. Functiorwl Arwlysis. New York: Ungar, 1955. 14. Saks, S. Theory of the Integral. (Monografje Matematyczne, Tom VII) ' Warszaw, Lwow, 1937.

 15. Stone, M. H. Lirmar Transformations in Hilbert Space. New York: American Mathematical Society, 1932.

16. Taylor, A. E. Introduction to Functional Arwlysis. New York: Wiley, 1958. 17. Zaanen, A. C. Lirmar Analysis. New York: Interscience, 1953.

Available online: Additional educational material is available on your page on the website univer.kaznu.kz in the EMSD section.

**Evaluation Criteria (Rating Scale):**

|  |  |  |  |
| --- | --- | --- | --- |
| **«Great» -** | А | 4,0 | 95-100 |
| А- | 3,67 | 90-94 |
| **«Fine» -**  | В+ | 3,33 | 85-89 |
| В | 3,0 | 80-84 |
| В- | 2,67 | 75-79 |
| С+ | 2,33 | 70-74 |
| **«Satisfactorily» -**  | С | 2,0 | 65-69 |
| С- | 1,67 | 60-64 |
| D+ | 1,33 | 55-59 |
| D- | 1,0 | 50-54 |
| **«Unsatisfactory» -**  | FX | 0,5 | 25-49 |
| F | 0 | 0-24 |

**GRADING POLICY**

**BAK/MAG/DOC STANDARD EXAM: ORAL**

|  |  |  |
| --- | --- | --- |
|  | **Criterion/score** | **Descriptors** |
|  | **Great** | **Fine** | **Satisfactorily** | **Unsatisfactory** |
| **№** | **90–100% (27-30 score)** | **70–89% (21-26 score)** | **50–69% (15-20 score)** | **25–49% (8-14 score)** | **0–24% (0-7 score)** |
| **1 question****30 points** | **Knowledge****and understanding****theories****and course****concepts** | “**Great**” grade is given for an answer that contains an exhaustive explanation of the question, a detailed argumentation for each conclusion and statement, is constructed logically and consistently, and is supported by examples from the developed classroom topics. | “**Fine**”grade is given for an answer that contains a complete but not exhaustive coverage of the issue, an abbreviated argumentation of the main provisions, and allows for a violation of the logic and sequence of presentation of the material. The answer contains inaccurate use of terms. | “**Satisfactory**” grade is given for an answer that contains incomplete coverage of the questions proposed in the ticket, superficially argues the main points, in the presentation allows for violations of the logic and sequence of presentation of the material, and does not illustrate theoretical points with examples from the developed class notes. | Incorrect coverage of the posed questions, erroneous argumentation, factual and verbal errors, assumption of an incorrect conclusion. | Ignorance of basic concepts, theories...; Violation of the Rules for final control. |
| **2 question****30 points** | **Application of favorites****methods and technologies****to specific****practical tasks** | Complete completion of the educational assignment, a detailed, reasoned answer to the question posed, followed by solving practical problems of the course; | Partial completion of the educational assignment, incomplete, sometimes reasoned answer to the question posed with an incomplete solution to the practical problems of the course; illiterate use of scientific language norms in the course; | The material is presented in fragments, in violation of logical sequence, factual and semantic inaccuracies are made, and theoretical knowledge of the course is used superficially. | An irrational method of solving a task or an insufficiently thought-out answer plan; inability to solve problems, perform tasks in general; making mistakes and omissions that exceeds the norm.  | Inability to apply knowledge and algorithms to solve tasks; inability to draw conclusions and generalizations. Violation of the Rules for final control. |

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|  | **Criterion/score** | **Descriptors** |
|  | **Great** | **Fine** | **Satisfactorily** | **Unsatisfactory** |
| **№** | **90–100% ((36-40 score) score)** | **70–89% (35-28 score)** | **50–69% ( (27-20 score)** | **25–49% (19-20 score)** | **0–24% (0-9 score)** |
| **3 question****40 points** | **Evaluating and analyzing the applicability of the chosen methodology to the proposed practical task, justifying the obtained result**  | Consistent, logical and correct justification of scientific principles and the applied methodology and technology, literacy, compliance with the norms of scientific language, 1-2 inaccuracies in the presentation of the material are allowed that do not affect the generally correct conclusions (+ visualization of the results of the justification through graphical data). | 3-4 inaccuracies in the use of conceptual material, minor errors in generalizations and conclusions are allowed, which do not affect the good overall level of task completion. | Conclusions on the applicability of substantiated scientific provisions are vague and unconvincing; there are stylistic and grammatical errors, as well as inaccuracies in processing the results of a practical solution | The task was completed with gross errors, the answers to the questions were incomplete, the conceptual material and argumentation were poorly used. | The task has not been completed, there are no answers to the questions posed, materials and analysis tools have not been used. Violation of the Rules for final control. |

Exam papers consist of 3 questions. For correctly completed tasks, the maximum is 100 points, of which 30 points for the first question, 30 **score** for the second question, and 40 **score** for the third question.